Pls state all the theorems that you are using irrespective of how simple it might be
The time limit for this paper is 3 hrs.
You are not allowed any documents or sheets with you and this is a totally closed book examination.
Question 1 is of 25 marks
Question 2 - 6 is of 35 marks each
Answer all questions.
Best of luck !!!

Full marks that can be earned is 200

1. Prove that the best approximation out of a finite dimensional subspace Y of the real vector space $C[a, b]$ is unique iff Y satisfies the Haar condition.

2. Consider the knot sequence for a spline (... 0, , 0, 1, ,1 ...). Such a knot sequence is used to create a Bezier polynomial from a B spline. Starting from the recurrence relation for the spline derive the recurrence for the Bezier

$$B_(\mu, \nu)(t) = tB_(\mu, \nu - 1)(t) + (1 - t)B_(\mu - 1, \nu)(t)$$
$$B_(\mu, \nu)(t) := B(t|000...00, 1111...11)$$

where the number of $0s$ is $\mu + 1$ and number of 1 s is $\nu + 1$

3. Prove that the Chebyshev Polynomials are orthogonal with the weight $(1 - x^2)^{-\frac{1}{2}}$

$$\int_{-1}^{1} T_n(x)T_m(x)\frac{dx}{\sqrt{1 - x^2}} = 0 \text{ if } m \neq n. \tag{1}$$

Hence prove the sequence $(T_k)_{k=0}^n$ forms a basis for the subspace P_n of $C[a, b]$

4. Can you use the error estimates of two point linear interpolation of the following function as $\frac{h^2}{8}\|f''\|$. Give a proof for your answer

$$f(x) = \sum_{1}^{\infty} 2^{-n} g(2^{2^n} x) \tag{2}$$

where $g(x) = 1 + x$ for $-2 \leq x \leq 0, g(x) = 1 - x$ for $0 \leq x \leq 2$

5. Using the two point formula and the four point formula for the Gaussian quadrature calculate the errors in the respective approximation of the integral $\int_0^4 te^{2t}dt$

6. Let Ω be an open subspace of R^2. Let K be a triangle $\subset \Omega$ Let the approximations of the function is done using linear ($P1$) elements. Assume that triangle K has vertices $(0,0),(h,0),(0,h)$ numbered 1,2,3. Then show that

$$a_{ij}^K = \int_k \bigtriangledown\phi_i \bigtriangledown \phi_j dx$$

using P1 approximation on the triangle K is

$$\begin{pmatrix} 1 & -0.5 & -0.5 \\ -0.5 & 0.5 & 0 \\ -0.5 & 0 & 0.5 \end{pmatrix}$$